On Some Applications of the Nakamura-Kuroda Theory of the Sedimentation of Colloid in a Vessel having Inclined Walls

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The increase of sedimentation velocity of a colloid in an inclined test tube was first reported by Boycott.(1) This curious phenomenon drew attention of many workers, and gave rise to the publication of many papers containing tentative theories to explain the phenomenon. Among these works the joint work of Nakamura and Kuroda might be considered as representative.(2) They made it clear that the inclined wall lying above the colloid (Fig. 1 a) gives rise to the increase of

(1) A. E. Boycott, *Nature*, **104**, 532 (1920).

(2) H. Nakamura and K. Kuroda, *Keijo J. Med.*, **8**, 256 (1937). [Keijo = Seoul, Korea].

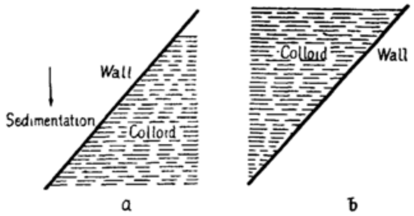


Fig. 1.

sedimentation velocity, while the wall situated below the colloid (Fig. 1 b) does not. From a very simplified assumption taking this fact into consideration, they gave a theory, though phenomenological in nature, which agrees with experiments quite well. Lately Kinosita examined the applicability of the Nakamura-Kuroda theory (abbreviated as N-K theory hereafter) in the settling of smokes and found that the theory holds as a whole.(3) In this paper, some results obtained by extending the fields of application of the N-K theory are given.

Sedimentation of Colloid in a Tube consisting of Two Parts having Different Sectional Area

A simple illustration of the N-K theory can be found in the sedimentation of colloid in a vessel consisting of two cylinders having different radii (Fig. 2). We know that the essential point of the N-K theory is expressed as follows.

Fig. 2.

$h \times$ upper boundary area of suspended phase
$= s \times$ horizontal projection of the upper boundaries of the suspension, (1)

where h = sedimentation velocity of colloid in a vessel having inclined walls,
s = sedimentation velocity of colloid determined in a vertical cylinder of uniform section.(4)

Applying the relation (1) in this case, we have

$$hA_1 = sA_2$$

or
$$h = s\,\frac{A_2}{A_1}, \tag{2}$$

where A_1 and A_2 are the sectional areas of the upper and lower cylinders. This relation shows that the initial sedimentation velocity of a colloid increases in a vessel having the form of Fig. 2, and that the amount of increase is determined by the ratio of the sectional areas of the lower and upper parts.(5)

Flotation of Emulsion in a Vessel under the Action of Oblique Centrifugal Force. Note on the Design of de Laval Centrifuge

It seems of interest to apply the N-K theory in a problem of centrifugal field. As an example, a problem of the design of de Laval centrifuge, which is usually used as cream separator, has been chosen. As is well known, the diagramatic sketch of the de Laval centrifuge is given in Fig. 3. Milk is kept in the spaces

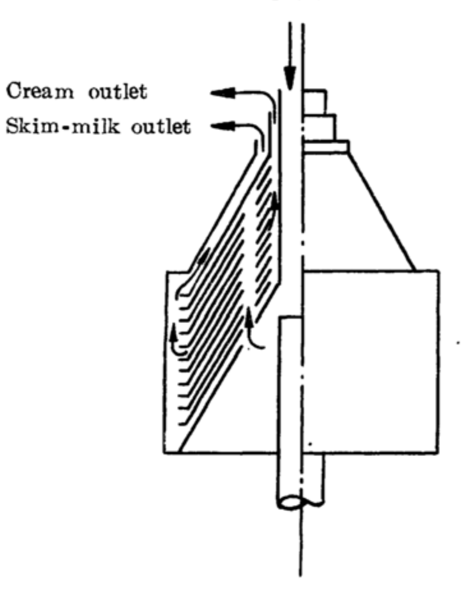


Fig. 3.

confined between two consecutive cones and is subjected to centrifugal force acting obliquely to the walls. In the actual case, the ratio between the slant height of cone and the

(3) K. Kinosita, *J. Colloid Sci.*, 4, 525 (1949).

(4) *cf.* (3) p. 526 below.

(5) A test carried out by Mr. T. Matuhasi, using suspension of blood corpuscle in physiological saline solution, showed that this relation actually holds.

distance between cones is about 120 and the inclination (the angle between the direction of centrifugal force and the slant height) is about 45°. A cursory glance at literature fails to find any study stating how these values were chosen. We shall examine whether the N-K theory is of use in interpreting these values.

In order to apply the N-K theory we neglect the flow of milk and observe only the flotation of fat particles which takes place under the action of centrifugal force. We assume that milk is an emulsion which is characterized by a flotation constant s_0 and confine our attention to the behavior of the milk contained in the space ABA'D'CD between the two cones ABA' and DCD' (Fig. 4). s_0 is a constant

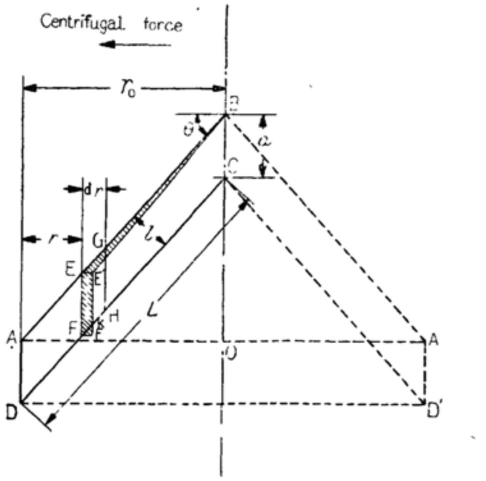


Fig. 4.

such that its product with centrifugal force gives the velocity of flotation. The conception of s_0 herewith is entirely in line with that of the sedimentation constant in ultracentrifuging.

Let us consider the milk in the parallelogram ABCD. The point A is taken as the origin of coordinate, and the distance from A toward the axis of rotation is taken as positive r. We denote the inclination by θ, and the angular velocity by ω. At first ABCD is uniformly filled with milk. With the elapse of time, flotation takes place and the boundary between skim-milk and cream moves toward the axis of rotation. We denote the boundary at the time t by EF, the distance of EF from the bottom AD by r, and the position of EF after the time interval dt by E'F'. From the definition of s_0, the distance EE' is given by $s_0\omega^2(r_0-r)\,dt$. The particles along EB should also move toward the axis of rotation with the elapse of time. From the fact that centrifugal force is proportional to the distance from the axis of rotation, the particles situated along EB should be found along E'B after the elapse of time dt. Thus the boundary after the time interval dt should be given by the line F'E'B. The small triangular area FF'K is neglected in the calculation. However, the actual boundary should remain always parallel to the axis of rotation. Following the principle of leveling action of the N-K theory, the position of the actual boundary GH is determined by the relation

$$\begin{aligned}2\pi\,(r_0-r)\,a\,dr &= 2\pi\,(r_0-r)\,a s_0\omega^2(r_0-r)\,dt \\ &+\left[\frac{1}{3}\pi(r_0-r)^2(r_0-r)\tan\theta-\frac{1}{3}\pi\left\{r_0-r\right.\right. \\ &\left.\left.-s_0\omega^2(r_0-r)\,dt\right\}^2(r_0-r)\tan\theta\right].\end{aligned}\tag{3}$$

This represents the condition that the volume generated by the revolution of the hatched area EFF'E'B is equal to the volume generated by the revolution of the area EFHG around the axis BO. Neglecting the terms containing the square of dt, and rearranging the remaining terms, we get

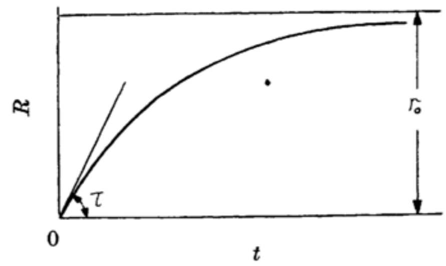
$$\frac{dr}{dt}=s_0\omega^2(r_0-r)\left\{1+\mu(r_0-r)\right\},$$

where $\mu=\dfrac{\tan\theta}{3a}$.

Integrating and making use of the initial condition, $r=0$ for $t=0$, we get

$$r=r_0-\frac{1}{\left(\mu+\dfrac{1}{r_0}\right)e^{s_0\omega^2 t}-\mu}.\tag{4}$$

This is the equation which gives the change of boundary with time. The graph of this equation is given in Fig. 5.



$\tan\tau=s_0\omega^2 r_0(1+r_0\mu)$

Fig. 5.

As the measure of the efficiency of cream separation, we take the initial velocity of flotation. We make $\left(\frac{dr}{dt}\right)_{t=0}$ from (4) and study its change with the inclination and the distance between the cones. By means of the relation $r_0 = L\cos\theta$, we get

$$\left(\frac{dr}{dt}\right)_{t=0} = s_0\omega^2 L\cdot\varphi(\alpha,\theta), \qquad (5)$$

where $$\varphi(\alpha,\theta) = \cos\theta(1+\alpha\sin 2\theta), \qquad (6)$$

$$\alpha = \frac{L}{6l}.$$

Thus we see that if s_0, ω and L are given, the initial velocity is determined by the function $\varphi(\alpha,\theta)$. In Fig. 6 the values of $\varphi(\alpha,\theta)$ are plotted against θ, taking α as parameter. The function $\varphi(\alpha,\theta)$ increases with the increase of α. If we denote by θ_m the value of θ which makes $\varphi(\alpha,\theta)$ maximum, it increases with the increase of α at first suddenly and then approaches the limiting value 35°.(6) The relation between α and θ_m is given in Fig. 7.

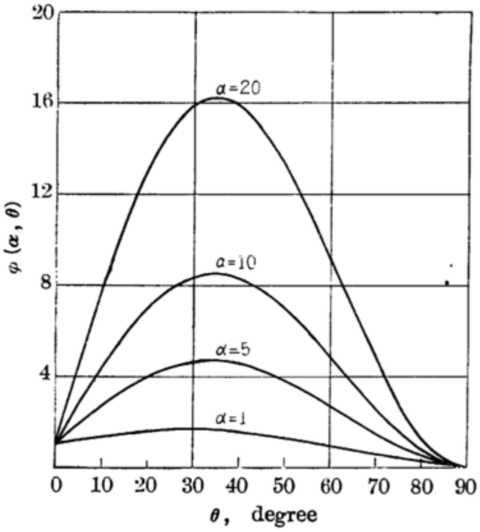


Fig. 6.

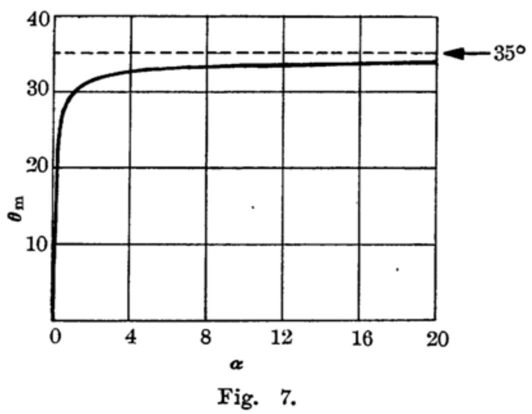


Fig. 7.

From the result obtained so far, larger value of α is desirable for higher efficiency. However, there is a limit for the value of α from the standpoint of mechanical construction, and we cannot increase it infinitely. The actual construction corresponds to the value 20 of α. The value of θ_m corresponding to α greater than 10 is nearly 35°, while the actual inclination is 45°—50°. The coincidence is not very good. However, so long as there is no reasonable way of determining the proper inclination, the conclusion obtained hereby might be of some use for the design of the centrifuge.

Summary

As a result of the application of the Nakamura-Kuroda theory of the sedimentation of colloid in an inclined cylinder it has been shown that the initial sedimentation velocity of colloid increases in a vessel consisting of two cylinders having different radii, the radius of the upper cylinder being smaller than that of the lower one. The amount of the increase is determined by the ratio of the sectional areas of the two cylinders.

The N-K theory has been formally extended in the flotation of emulsion in a centrifugal field, and the design of the de Laval centrifuge has been considered in the light of the results obtained. A feasible, though not quite rigorous, explanation of the consistency of the form of centrifuge has been given.

In conclusion, the author would like to express his cordial thanks to Professor S. Oka for his kind interest in this work.

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(6) This is the value of θ which makes $\varphi(\alpha,\theta)$ maximum when α is very great. It is obtained by solving $\frac{d\varphi}{d\theta}=0$, neglecting the term which does not contain α.